

CHAPTER NINE

Liquid Mixing

9.1 Introduction

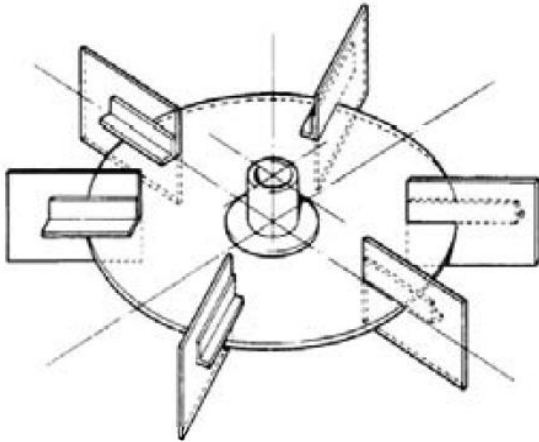
Mixing is one of the most common operations carried out in the chemical, processing. The term “ Mixing” is applied to the processes used to reduce the degree of non-uniformity, or gradient of a property in the system such as concentrations, viscosity, temperature, and so on. Mixing is achieved by moving material from one region to another. It may be interest simply as a means of achieving a desired degree of homogeneity but it may also be used to promote heat and mass transfer, often where a system is undergoing a chemical reaction.

A rotating agitator generates high velocity streams of liquid, which in turn entrain stagnant or slower moving regions of liquid resulting in uniform mixing by momentum transfer. As viscosity of the liquid is increased, the mixing process becomes more difficult since frictional drag retards the high velocity streams and confines them to immediate vicinity of the rotating agitator.

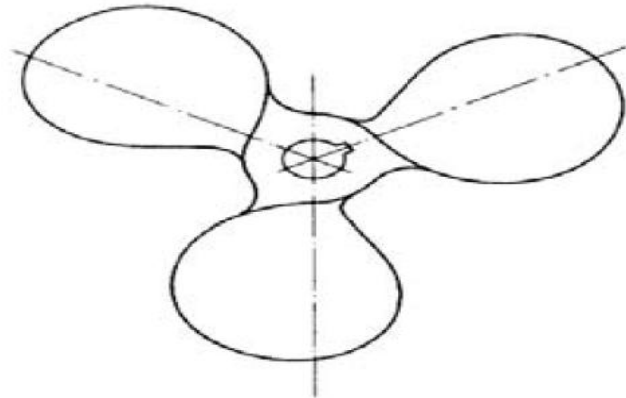
9.2 Types of Agitators

In general, agitators can be classified into the following two types: -

- 1- Agitators with a small blade area, which rotate at high speeds. These include **turbines and marine type propellers.**
- 2- Agitators with a large blade area, which rotate at low speeds. These include **anchors, and Paddles, and helical screws.**



Six-blade flat blade turbine



Marine Propeller

The second group is more effective than the first in the mixing of high viscosity-liquids.

For a liquid mixed in a tank with a rotating agitator, the shear rate is greatest in the intermediate vicinity of agitator. In fact the shear rate decreases exponentially with distance from the agitator. Thus the shear stresses and strains vary greatly throughout an agitated liquid in tank. Since the dynamic viscosity of a Newtonian liquid is independent of shear rate at a given temperature, its viscosity will be the same at all points in the tank. In contrast the apparent viscosity of a non-Newtonian liquid varies throughout the tank. This in turn significantly influences the mixing process.

The mean shear $\dot{\gamma}_m$ produced by an agitator in a mixing tank is proportional to the rotational speed of the agitator N

i. e. $\dot{\gamma}_m \propto N \Rightarrow \dot{\gamma}_m = KN$

where, K is a dimensionless proportionality constant for a particular system. It is desirable to produce a particular mixing result in the minimum time (t) and with the minimum input power per unit volume (PA/V).

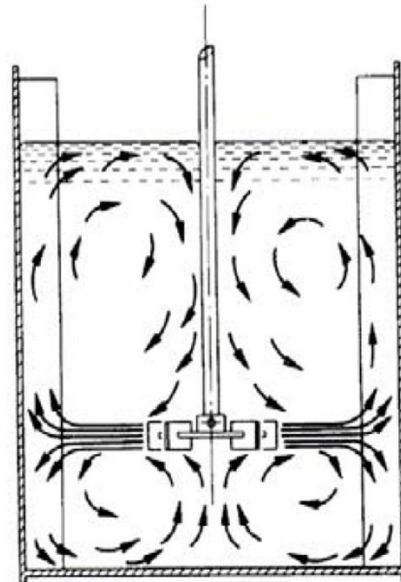
Thus the efficiency function (E) can be defined as $E = \left(\frac{1}{P_A/V}\right) \left(\frac{1}{t}\right)$

9.2.1 Small Blade, High Speed Agitators

They are used to mix low to medium viscosity liquids. Two of most common types are 6-blade flat blade turbine and the marine type propeller.

Flat blade turbines used to mix liquids in baffled tanks produce radial flow patterns primarily perpendicular to the vessel wall. This type is suitable to mix liquids with dynamic viscosity up to 50 Pa.s.

Marine type Propellers used to mix liquids in baffled tanks produce axial flow patterns primarily parallel to the vessel wall. This type is suitable to mix liquids with dynamic viscosity up to 10 Pa.s.



Radial flow pattern produced by a flat blade turbine

Agitator Tip Speed (TS)

Is commonly used as a measure of the degree of the agitation in a liquid mixing system.

$$T_S = \pi D_A N$$

Where, D_A : diameter of agitator.

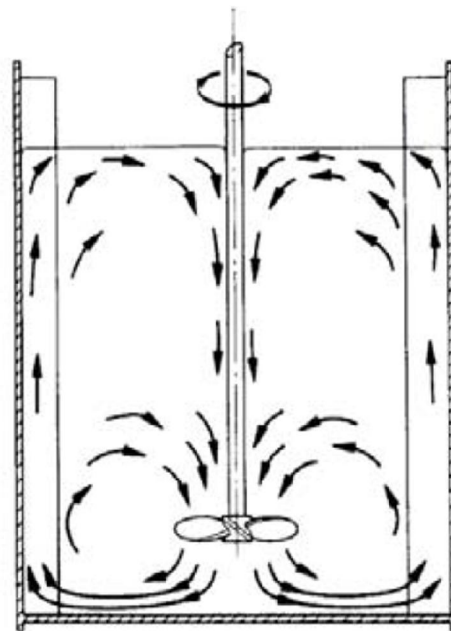
N: rotational speed.

Tip speed ranges for turbine agitator are recommended as follows:

$T_S = 2.5$ to 3.3 m/s for low agitation.

$T_S = 3.3$ to 4.1 m/s for medium agitation.

$T_S = 4.1$ to 5.6 m/s for high agitation.



Axial flow pattern produced by a marine

Standard Tank Configuration

A turbine agitator of diameter (D_A) in a cylindrical tank of diameter (D_T) filled with liquid to a height (H_L). The agitator is located at a height (H_A) from the bottom of the tank and the baffles, which are located immediately adjacent to the wall, have a width (b). The agitator has a blade width (a) and a blade length (r) and the blades are mounted on a central disc of diameter (s).

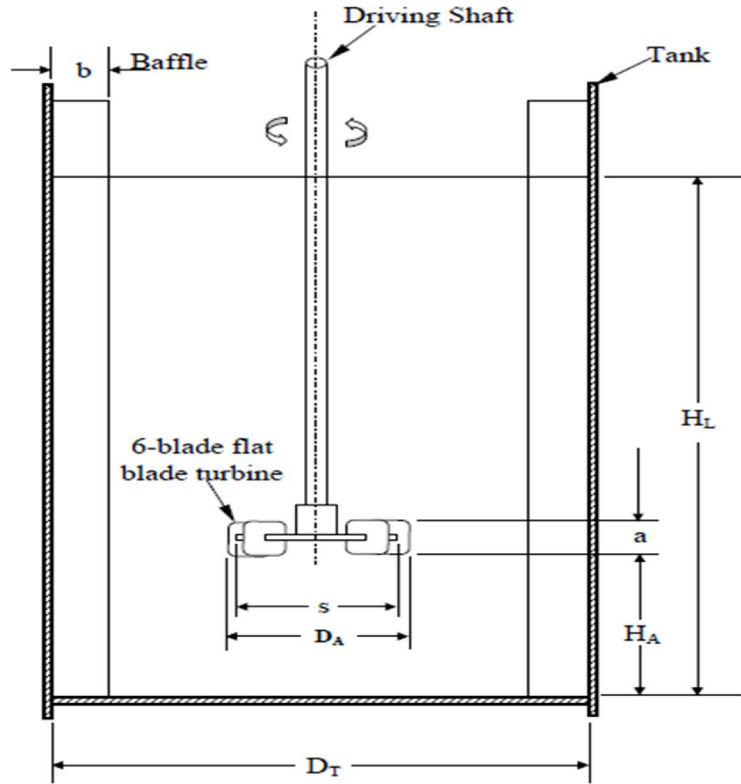
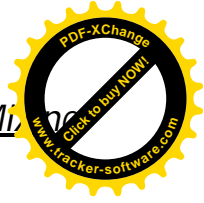
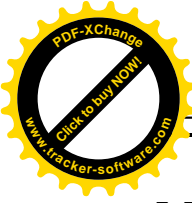


Figure of Standard Tank Configuration

A typical turbine mixing system is the standard configuration defined by the following geometrical relationships: -

- 1- a 6-blade flat blade turbine agitator.
- 2- $D_A = D_T / 3$
- 3- $H_A = D_T / 3$
- 4- $a = D_T / 5$
- 5- $r = D_T / 8$
- 6- $H_L = D_T$
- 7- 4 symmetrical baffles
- 8- $b = D_T / 10$

Processing considerations sometimes necessitate deviations from the standard configuration.



Marine Type Propeller

It can be considered as a case-less pump. In this case its volumetric circulating capacity (Q_A) is related to volumetric displacement per revolution (V_D) by the equation;

$$Q_A = \eta V_D N$$

where, η : is a dimensionless efficiency factor which is approximately (0.6).

V_D is related to the propeller pitch (P) and the propeller diameter (D_A) by the equation;

$$V_D = \frac{\pi D_A^2 P}{4}$$

Most propellers are square pitch propellers where ($P = D_A$) so that the last equation becomes;

$$V_D = \frac{\pi D_A^3}{4} \Rightarrow Q_A = \frac{\eta \pi D_A^3 N}{4}$$

A tank turnover rate (IT) is defined by the equation;

$$I_T = Q_A / V$$

where, V: is the tank volume and I_T : is the number of turnovers per unit time.

To get the best mixing I_T should be at a maximum for a given tank volume (V), this means that the circulating capacity Q_A should have the highest possible value for the minimum consumption of power.

The head developed by the rotating agitator (h_A) can be written as;

$$h_A = C_1 N^2 D_{A2} \text{ where, } C_1 \text{ is a constant.}$$

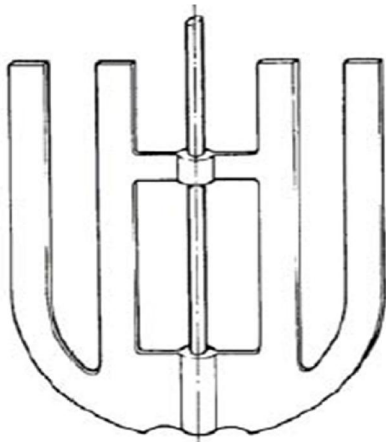
$$Q_A/h_A = C D_A/N \text{ where, } C = \eta\pi/(4C_1)$$

$$\text{but } \dot{\gamma}_m = KN$$

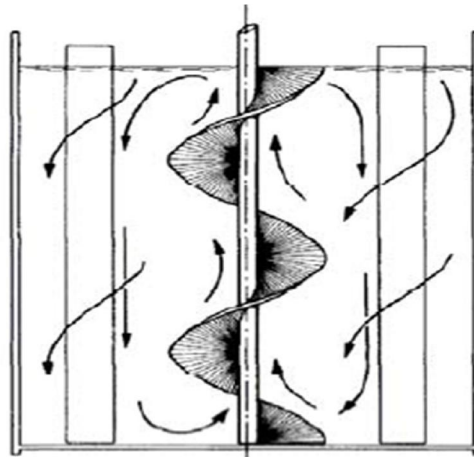
$$\frac{Q_A}{h_A} = C' \frac{D_A}{\dot{\gamma}_m} \quad \text{where } C' = C.K = \text{constant}$$

9.2.2 Small Blade, High Speed Agitators

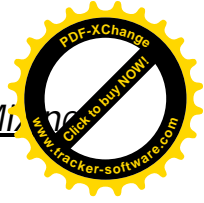
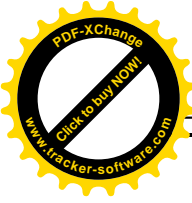
This type of agitators includes anchors, gates, paddles, helical ribbons, and helical screws. They are used to mix relatively high viscosity liquids and depend on a large blade area to produce liquid movement throughout a tank. Since they are low shear agitators.



Gate type anchor agitator



Flow pattern in a baffled helical screw system



9.3 Dimensionless Groups for Mixing

Some of the various types of forces that may be arise during mixing or agitation will be formulated: -

1- Inertial Force [Fi]

Is associated with the reluctance of a body to change its state of rest or motion.

The inertial force (Fi) = (mass) (acceleration) = m.a

$$dF_i = dm (du/dt) \text{ but } m = \rho V = \rho A L \Rightarrow dm = \rho dV = \rho A dL$$

$$\text{and } u = dL/dt \Rightarrow dF_i = \rho A dL du/dt = \rho A (dL/dt) du = \rho A u du$$

$$\Rightarrow F_i = \int_0^u dF_i = \int_0^u \rho A u du = \rho A u^2 / 2$$

$$\text{or } \dot{m} = \frac{dm}{dt} \Rightarrow dm = \dot{m} dt = \rho A u dt$$

In mixing applications;

$$A \propto D_A^2 \quad D_A: \text{ diameter of agitator}$$

$$u = \pi D_A N \quad N: \text{ rotational speed}$$

Therefore, the expression for inertial force may be written as;

$$F_i \propto \rho D_A^4 N$$

2- Viscous Force [Fv]

The viscous force for Newtonian fluid is given by:

$$F_v = \mu A (du/dy) \text{ In mixing applications;}$$

$$A \propto D_A^2; du/dy \propto \pi D_A N / D_A$$

Therefore, the expression for viscous force may be written as;

$$F_v \propto \mu D_A^2 N$$

3- Gravity Force [Fg]

The inertial force (Fg) = (mass) (gravitational acceleration) = m.g

$$\text{In mixing applications; } m = \rho V = \rho A L \propto \rho D_A^3$$

$$F_g \propto \rho D_A^3 g$$

4- Surface Tension Force [Fσ]

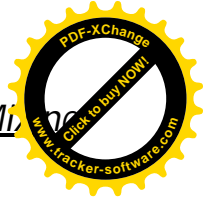
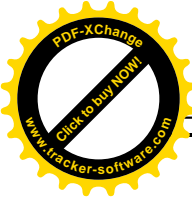
In mixing applications; $F_\sigma \propto \sigma DA$

In the design of liquid mixing systems the following dimensionless groups are of importance: -

1- The Power Number (Np)

$$N_p = P_A / \rho N^3 D_A^5$$

where, P_A : is the power consumption.



2- The Reynolds Number (Re)_m

$$(Re)_m = \frac{\text{Inertial Force}}{\text{Viscous Force}} = \frac{F_i}{F_v} = \frac{\rho D_A^4 N^2}{\mu D_A^2 N} \Rightarrow (Re)_m = \frac{\rho D_A^2 N}{\mu}$$

3- The Froude Number (Fr)_m

This number related to fluid surface [related to vortex system in mixing]

$$(Fr)_m = \frac{\text{Inertial Force}}{\text{Gravity Force}} = \frac{F_i}{F_g} = \frac{\rho D_A^4 N^2}{\rho D_A^3 g} \Rightarrow (Fr)_m = \frac{\rho D_A^4 N^2}{g}$$

4- The Weber Number (We)_m

This number related to multiphase fluids [or fluid flow with interfacial forces]

$$(We)_m = \frac{\text{Inertial Force}}{\text{Surface Tension Force}} = \frac{F_i}{F_g} = \frac{\rho D_A^4 N^2}{\sigma D_A} \Rightarrow (We)_m = \frac{\rho D_A^3 N^2}{\sigma}$$

It can be shown by dimensional analysis that the power number (N_p) can be related to the Reynolds number (Re)_m and the Froude number (Fr)_m by the equation;

$$N_P = C(Re)_m^x (Fr)_m^y$$

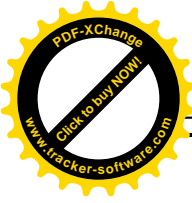
where, C is an overall dimensionless shape factor which represents the geometry of the system.

The last equation can also be written in the form;

$$\Phi = \frac{N_P}{(Fr)_m^y} = C(Re)_m^x$$

where, ϕ is defined as the dimensionless power function.

The Froude number (Fr)_m is usually important only in situations where gross vortexing. Since vortexing is a gravitational effect, the (Fr)_m is not required to describe a baffled liquid mixing systems.



In this case the exponent of $(Fr)_m$ (i.e. y) in the last two equations is zero. $[(Fr)^y = (Fr)^0 = 1 \Rightarrow \phi = N_p$

Thus the non-vortexing systems, the equation of power function (ϕ) can be written wither as;

$$\phi = N_p = C(Re)_m^x \quad \text{or as; } \log \phi = \log N_p = \log C - x \log(Re)_m$$

The Weber number of mixing $(We)_m$ is only of importance *when separate physical phases are present in the liquid mixing system as in liquid-liquid extraction.*

9.4 Power Curve

A power curve is a plot of the power function (ϕ) or the power number (N_p) against the Reynolds number of mixing $(Re)_m$ on log-log coordinates. Each geometrical configuration has its own power curve and since the plot involves dimensionless groups it is independent of tank size. **Thus a power curve that used to correlate power data in a 1 m³ tank system is also valid for a 1000 m³ tank system** provided that both tank systems have the same **geometrical configuration.**

The Figure below shows the power curve for the standard tank configuration. Since this is a baffled tank (non-vortexing system), the following equation is applied;

$$\log \phi = \log N_p = \log C - x \log(Re)_m \text{-----} (*)$$

which can be rearranged to give;

$$P_A = C \mu N^2 D_A^3$$

C is a constant depend on the type of agitator and vessel arrangement and if the tank is with or without baffles. **For the standard tank configuration $C = 71$ and for marine type 3-blade $C = 41$.** Thus for the laminar flow, power (P_A) is directly proportional to *dynamic viscosity (μ) for a fixed agitator speed (N)*.

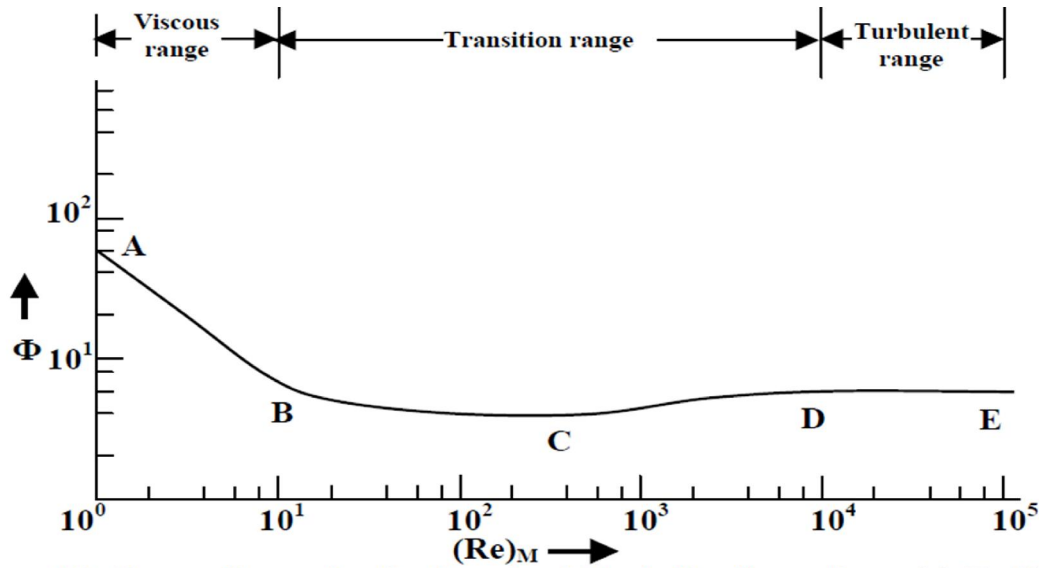
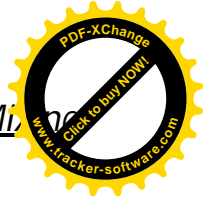
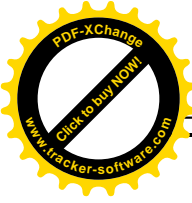


Figure (1): Power Curve for the Standard Tank Configuration with Baffles



The power curve for the baffled system is identical with that for the unbaffled system up to point (C) where $[(Re)_m \approx 300]$. As the $(Re)_m$ increases beyond point (C) in the unbaffled system, vortexing increases and the power falls sharply as shown in the above Figure.

As mentioned previously it can be shown by **dimensional analysis** that the **power number (N_p) can be related to the Reynolds number $(Re)_m$ and the Froude number $(Fr)_m$ by the equation;**

$$N_p = C(Re)_m^x (Fr)_m^y$$

$$\Rightarrow \log N_p = \log C - x \log (Re)_m - y \log (Fr)_m$$

For the un baffled system

$$\Phi = N_p \quad \text{for } (Re)_m < 300$$

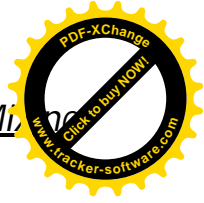
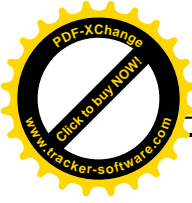
$$\Phi = N_p / (Fr)_m^y \quad \text{for } (Re)_m > 300$$

A plot of (N_p) against $(Fr)_m$ on log-log coordinates is a straight line of slope y at a constant $(Re)_m$. A number of lines can be plotted for different values of $(Re)_m$. A plot of (y) against $\log(Re)_m$ is also a straight line. If the **slope of the line is $(-1/\beta)$ and the intercept at $(Re)_m = 1$ is (α/β)** then

$$y = \frac{\alpha - \log (Re)_m}{\beta}$$

$$\Phi = \frac{N_p}{(Fr)_m^y} = \frac{N_p}{[(Fr)_m]^{\frac{\alpha - \log (Re)_m}{\beta}}}$$

The values of $(\alpha$ and $\beta)$ are varying for various vortexing system. For a 6-blade flat blade turbine agitator of 0.1 m diameter $[(\alpha = 1)$ and $(\beta = 40)]$



If a power curve is available for particular system geometry, it can be used to calculate the power consumed by an agitator at various rotational speeds, liquid viscosities and densities. The procedure is as follows: -

- 1- Calculate $(Re)_m$
- 2- Read power number (N_p) or power function (Φ) from the appropriate power curve
- 3- Calculate the power (P_A) from

$$P_A = N_P \rho N^3 D_A^5 \quad \text{or} \quad P_A = \Phi [(Fr)^y] \rho N^3 D_A^5$$

These equations can be used to calculate only the power consumed by the agitator. Electrical and mechanical losses required additional power, which occur in all mixing system.

Example -9.1-

Calculate the theoretical power in Watt for a 3 m diameter, 6-blade flat blade turbine agitator running at 0.2 rev/s in a tank system conforming to the standard tank configuration. The liquid in the tank has a dynamic viscosity of 1 Pa.s, and a liquid density of 1000 kg/m³.

Solution:

$$(Re)_m = \rho N D_A^2 / \mu = (1000) (0.2) (3)^2 / 1 = 1,800$$

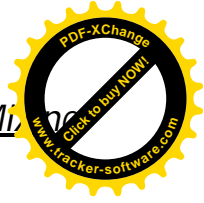
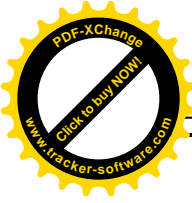
From Figure (1) $\Phi = N_p = 4.5$

$$P_A = N_P \rho N^3 D_A^5$$

The theoretical power for mixing

$$= 4.5 (1000) (0.2)^3 (3)^5$$

$$= 8,748 \text{ W}$$

**Example -9.2-**

Calculate the theoretical power in Watt for a 0.1 m diameter, 6-blade flat blade turbine agitator running at 16 rev/s in a tank system without baffles and conforming to the standard tank configuration. The liquid in the tank has a dynamic viscosity of 0.08 Pa.s, and a liquid density of 900 kg/m³.

Solution:

$$(Re)_m = \frac{\rho D_A^2 N}{\mu} = (900) (16) (0.1)^2 / (0.08) = 1,800$$

From Figure (2) $\Phi = 2.2$. The theoretical power for mixing

$$P_A = \Phi [(Fr)^y] \rho N^3 D_A^5$$
$$y = \frac{\alpha - \log (Re)_m}{\beta} \Rightarrow y = \frac{1 - \log (1800)}{40} = -0.05638$$

$$(Fr)_m = N^2 D_A / g = (16)^2 (0.1) / 9.81 = 2.61$$

$$[(Fr)_m]^y = [2.61]^{-0.05638} = 0.9479$$

$$\Rightarrow P_A = 2.2 (0.9479) (900) (16)^3 (0.1)^5 = 76.88 \text{ W}$$

